# Extended Abstract: Managing Disjunction for Practical Temporal Reasoning

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## Abstract

One of the problems that must be dealt with in either a formal or implemented temporal reasoning system is the ambiguity arising from uncertain information. Lack of precise information about when events happen leads to uncertainty regarding the effects of those events. Incomplete information and nonmonotonic inference lead to situations where there is more than one set of possible inferences, even when there is no temporal uncertainty at all. In an implemented system, this ambiguity is a computational problem as well as a semantic one.

In this paper, we discuss some of the sources of this ambiguity, which we will treat as explicit *disjunction*, in the sense that ambiguous information can be interpreted as defining a set of possible inferences. We describe the application of three techniques for managing disjunction in an implementation of Dean's *Time Map Manager*. Briefly, the disjunction is either: removed by limiting the expressive power of the system, explicitly represented, one disjunct at a time, or approximated by a weaker form of representation that subsumes the disjunction. We use a combination of these methods to implement an expressive and efficient temporal reasoning engine that performs sound inference in accordance with a well-defined formal semantics.

## 1 Introduction

One of the problems that must be dealt with in either a formal or implemented temporal reasoning system is the disjunction arising from uncertain information. Lack of precise information about when events happen leads to uncertainty regarding the effects of those events, and thus to uncertainty in what propositions are true at some point in time. Incomplete information regarding what propositions are true when, and nonmonotonic inference (e.g., the persistence assumption or qualified causal projection) lead to situations where there is more than one set of possible inferences, even when there is no temporal uncertainty at all [6]. In a formal system, this ambiguity is noted and in some way dealt with, either by changing the semantics to exclude it (e.g. by assigning a preference relation to the possible models of a given theory), or simply by acknowledging it (i.e. couching conclusions in terms of the *set* of possible models).

In an implemented system, this ambiguity is a computational problem as well as a semantic one. In this paper, we discuss some of the sources of this ambiguity, which we will treat as explicit *disjunction*, in the sense that ambiguous information can be interpreted as defining a set of possible inferences. We describe how these sources of disjunction are dealt with in our current implementation of Dean's *Time Map Manager* [5; 2]. Briefly, we take one of three approaches:

1. The disjunction is removed by limiting the expressive power of the system.
2. The disjunction is explicitly treated, but the system considers only a single disjunct at a time.
3. The disjunction is approximated by a weaker form of representation that subsumes the disjunction.

The semantics that we are attempting to capture in our implementation are defined in [1], which provides a precise formal semantics for the current version of the TMM.

In the rest of this paper, we briefly discuss the ontology and semantics of the TMM, provide some specific examples of the kinds of disjunction that arise, and discuss the costs and benefits of various ways of handling these types of disjunction.

## 2 The TMM

Dean's *Time Map Manager* [5; 2] is an implemented temporal reasoning system, intended as a foundation for building planning and scheduling systems. The TMMincludes capabilities for reasoning about partially-ordered events, persistence and clipping, and two simple forms of causal reasoning: projection and temporal implication (sometimes called "overlap chaining" in previous work). The version of the system described in [5; 2] that was distributed from Brown (hereinafter referred to as "$\alpha$-TMM") implements forward persistence only, and does not implement temporal implication.

Besides these limitations, the inference performed by $\alpha$-TMM is not sound for partially-ordered time points [3], and so has no well-defined semantics. For partial orders, the inference done by the system is interpreted as quantification over total orders consistent with a given partial order: a formula of the form holds(t, P) is interpreted to mean that the proposition P holds at the time point t in all possible total orders. The sense in which the original system is unsound is that it will sometimes infer holds(t, P) when there were total orders in which P does not hold at t. As Dean and Boddy show in the same paper, reasoning about what is true in the total orders consistent with a given partial order is an NP-complete problem.

We have addressed these difficulties by implementing a sound but incomplete decision procedure that approximates quantification over time points (*i.e.*, if the system infers holds(t, P), the proposition P does in fact hold at the

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time point t in every total order, but sometimes this property will be true and the system will not infer holds(t, P) [4]. We have made other extensions, including generalizing persistence to run backward as well as forward (in order to handle cases like Kautz's "parking lot problem." [7]), and implementing *temporal implication:* reasoning in which the truth of some set of facts at a point can be used to conclude that some other fact is true at the same point. We have retained from the old system the concepts of *persistence clipping* and *causal projection* (referred to hereinafter as simply "projection").[1] The new TMM implementation we will refer to as "$\beta$-TMM."

As far as we know, $\beta$-TMM is the first implementation of sound-and-incomplete temporal reasoning as described in [4]. The process of implementing this decision procedure has made clear precisely how the resulting system is incomplete; this point will be addressed in Section ??.

## 2.1 Ontology and Inference

In this section we present a simplified version of the TMM representations that is sufficient for this discussion. A *domain theory* in the language includes a *time map* and a *causal theory.* The time map consists of a set of *time points* $\mathcal{T}$ and a set of formulas. Time map formulas include the following:

- *Temporal relations* between time points, denoted by the binary infix predicates $<$, $\leq$, $=$, $\geq$, and $>$, and the predicate distance(t1, t2, *bounds*), where $t1, t2 \in \mathcal{T}$ and *bounds* = [r1 r2] where r1, r2 $\in \Re$ are the bounds of a closed interval. We represent temporal relations in the time map as *constraints.*
- *Temporal formulas,* holds(t1, t2, P), where $t1, t2 \in \mathcal{T}$ and $P \in \mathcal{P}$, the set of *propositions.* The period between t1 and t2 is called the "observation interval" (throughout which the proposition must necessarily hold.) We use the abbreviation holds(t, P) when this interval is a point. We represent temporal formulas on the time map using *time tokens.*
- *Persistence assumptions,* $\text{persists}_f$(t1, P) and $\text{persists}_b$(t2, P), where t1, t2, and P appear in some temporal formula as above. We associate persistence assumptions with time tokens on the time map.

The causal theory for a TMM theory includes *causal rules,* intended to encode the physics of a domain in a simple way, of the following kinds.

- *Projection rules,* project((and $\langle P_1, \ldots, P_k \rangle$), E, R). The propositions $P_1, \ldots, P_k$ are *antecedents*; E is a "trigger" proposition; R a forward-persistent "result" proposition. When the antecedent propositions are believed to hold throughout the trigger, the result is believed starting at a specified time after the trigger.
- *Temporal implication rules,* (and $\langle P_1, \ldots, P_k \rangle$) $\Rightarrow_t$ R. At any point for which the propositions of the antecedent conjunction are all believed to hold, the result proposition R is believed to hold.

[1] Details of extensions planned and accomplished can be obtained by request from Bob Schrag, at the address at the beginning of this paper.

The TMM implements an epistemic semantics, in the sense that a proposition may be known (or believed) to hold at a point, or known not to hold at that point, or we may not know either way. This semantics is described more carefully in [1]. The failure of the excluded middle in this semantics is useful for representing problems where we have only partial information. All of the propositions in the domain theory are believed necessarily. Temporal propositions are believed necessarily at all points throughout their observation intervals. Inference from projection and temporal implication result in the addition of new tokens to the time map, representing belief in propostions holding for new intervals of time. Persistence is captured in a preference over models: those in which the appropriate facts persist are preferred over those in which they don't. Conflicts in these preferences result in ambiguous situations, where no single set of inferences can be preferred to all others.

The theory including the time map and causal rules is intended to support the following kinds of inference.

- holds(t1, t2, P): P is true in all possible worlds.
- $\text{holds}_m$(t1, t2, P): P is true in some possible world.
- Inferences about necessary and possible temporal relations.
- Boolean combinations of these.

The first two kinds of inference concern belief in quantifications of temporal formulas over possible worlds consistent with the user-supplied domain theory. The simplest form of ambiguity in the domain theory that can lead to multiple possible worlds results from a set of temporal relations that defines only a partial order on the set of time points.

## 2.2 Sources of Disjunction

There are several sources of disjunction in the TMM. There is one source of disjunction we have explicitly removed: there is no way to assert an explicit disjunction in the domain theory. You can say that proposition P is true at time t, and that point t1 is ordered before point t2. You cannot, for example, say that t1 and t2 cannot occur simultaneously (*i.e.,* they are definitely ordered one way or the other).

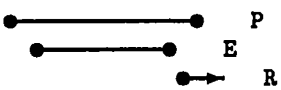
This leaves us with two main classes of disjunction to deal with. The first is the temporal uncertainty resulting from the fact that we do not require time points to be totally ordered. Actually, there is additional metric uncertainty: we can specify the distance between two time points only as a range without that meaning that there is any uncertainty in ordering anywhere in the time map. Metric temporal uncertainty is straightforward to deal with. It affects no inference more complicated than directly determining whether a proposition holds at a point. Partially ordered points are a more complex problem because ordering affects which inference rules fire. For either projection or temporal implication, whether the rules fire is based solely on ordering relationships: all the possible assignments to temporal relations consistent with a given total order are equivalent, as far as which causal rules will "fire." For this source of disjunction, the "possible worlds" are the total orders consistent with the given partial order. Deciding whether a proposition holds at a point necessarily, possibly, or not at all becomes a question of quantifying over the set of total orders. In Section 3, we discuss how this is accomplished (approximated, actually) in the TMM.

The other source of disjunction we must consider is a direct result of the semantics we impose on the system: the persistence assumption. Nonmonotonic reasoning has been recognized by many people at many times as a source of ambiguity and unintended conclusions (most relevant to our work is Hanks and McDermott's paper on applying nonmonotonic logic to temporal reasoning [6]). Unfortunately, it appears to be too useful to dispense with. Simply stated, the persistence assumption says that things tend not to change unless something changes them. If I walk into a room, see that the light is on, and walk out again, it seems both reasonable and useful to conclude that the light was on before I got there, and again after I left. [2] Contradictory information (*e.g.,* walking into the room at a later point and noticing that the light is off) will cause the system to draw different conclusions. The persistence assumption can lead to ambiguous conclusions in a wide variety of situations, a representative sampling of which are discussed in Section 4.

In the examples in the following sections, we represent time maps as follows: A time point is represented by a dot: ●. An observation interval is represented by two time points connected by a line: ●——●. Temporal ordering is from left to right, and all points are drawn with respect to a given frame of reference. When a time point is connected to a solid line, we know its relation with respect to the reference exactly. A dashed line as in ●----•——● indicates uncertainty about the point's location. Forward and backward persistence are represented by forward- and backward-pointing arrows: ◄—, —►. We label tokens with the corresponding propositions and we label time points when we need to refer to them: ● t1. A lone timepoint with a proposition label is a zero-length observation interval: ● P. A single time point with a persistence symbol is a persistent version of the same thing: ●—► P.

To illustrate, here is a simple time map situation demonstrating the firing of a projection rule. Relevant textual information is displayed above the time map.

project(P, E, R)




## 3 Partial Orders

The problem with partially-ordered time maps is that inference such as projection and temporal implication depend on what facts hold at a given point. This relation is defined only for totally ordered points, and so we are reduced to determining what facts might possibly or necessarily hold at a point, in some or all of the total orders consistent with the given partial order. With even a very simple causal model, this is an NP-complete problem [4]. The solution we have implemented (first presented in [3]) is to approximate the necessary quantification.

$\beta$-TMM includes two holds definitions which together provide a sound-and-incomplete temporal reasoning algorithm which executes in polynomial time. Each definition approximates a quantification over the possible worlds consistent with the domain theory. holds$_S$ (***strong*** holds) is a sound-and-incomplete approximation to holds. We use holds$_S$ to identify a ***subset*** of all necessarily believed temporal propositions. holds$_W$ (***weak*** holds) is a complete-and-unsound approximation to holds$_m$. We use holds$_W$ to identify a ***superset*** of all possibly believed temporal formulas. In the presence of inference such as projection, the strong version requires the weak version: a proposition necessarily holds over an interval unless there is a possibly-derived token (the result of a projection rule, or added by the user), which possibly contradicts (clips) that proposition for some part of that interval.

holds$_S$ is ***incomplete*** in two ways:

- It avoids combinatorics by looking for a single token to span the query interval for ***all*** possible worlds. It will fail in a case where the interval is spanned by different tokens in different total orders.
- It relies, ultimately, on the over-achieving holds$_W$ to defeat the strong tokens' persistences.

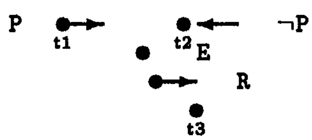
holds$_W$ is ***unsound*** in two ways:

- It avoids combinatorics by checking for a conjunction of possibilities rather than a possible conjunction. It succeeds sometimes when the conjuncts are not mutually satisfiable.
- It relies, ultimately, on the under-achieving holds$_S$ to defeat the weak tokens' persistences.

Some of these points are illustrated in the following examples.

**Example 1:** Incompleteness in holds$_S$ can arise directly from opposing contradictory persistences.

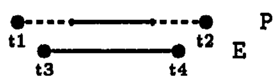
project(P, E, R)
project(¬P, E, R)




Our semantics says that the persistences for P and ¬P clip at some point between t1 and t2l, but not where. One of P or ¬P covers E in all total orders, so holds(t3, R). We are limited to holds$_W$(t3, R).

**Example 2:** Unsoundness in holds$_W$ can arise directly from partially ordered timepoints.

distance(t1, t2, 3)
distance(t3, t4, 5)



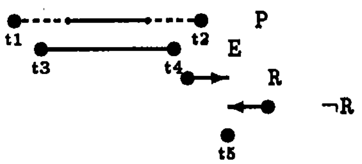

P does

[2]How "reasonable" persistence is, is context-dependent. Consider the same example where I see a cat sleeping on a chair, or a newspaper on a seat on a train.

not cover E in any possible world, so $\neg$holds$_m$(t3, t4, P)—but the conjunction of possible temporal relations in holds$_w$(t3, t4, P) is satisfied, and it succeeds, unsoundly. Even though we do not have $(t1 \leq t3 \wedge_m t2 \geq t4)$, we do have $(t1 \leq_m t3 \wedge t2 \geq_m t4)$.

**Example 3:** Incompleteness in holds$_s$ can arise indirectly, through weakly and unsoundly derived defeaters.

distance(t1, t2, 3)
distance(t3, t4, 5)
project(P, E, R)




From Example 2 above, we know the token for R is weakly and unsoundly derived, and we should have holds(t7, $\neg$R). But $\neg$R is defeated weakly and unsoundly and we are limited to holds$_w$(t7, $\neg$R).

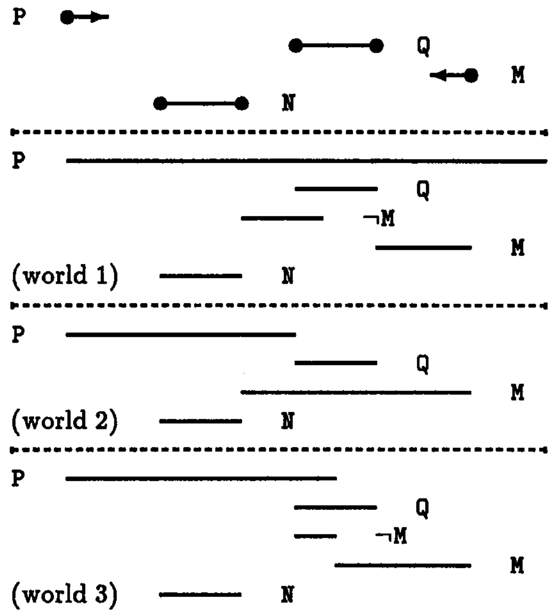
While strong inference (holds$_s$) is incomplete in a well-defined and limited sense (checking a single token), the approximate nature of weak inference (holds$_w$) is less precise. There are tradeoffs that can be made. For example, it is possible to add or omit a check on the maximum possible extent of a given token, rather than just the ordering of the endpoints. Adding such a check would result in a system that handled Example 2 correctly. At an additional computational expense, of course.

# 4 Ambiguous Models Resulting From Persistence

The persistence assumption combines with temporal implication or projection to generate situations in which there are several possible models for a given domain theory. In other words, we can construct theories in which P is true at some time T in some models (possible worlds) and false in others. These situations arise even if we limit ourselves to theories where all temporal relations are precisely specified for every point in the time map. In the following scenario, there are two temporal implication rules and four tokens specified in the domain theory (the dashed line on the right hand side separates a picture of the initial conditions from three different "possible worlds" corresponding to different models that can be constructed).

**Example 4:** Temporal implication with persistence can be ambiguous.

R1: (and P Q) $\Rightarrow_t$ $\neg$M
R2: (and M N) $\Rightarrow_t$ $\neg$P




In the first possible world (below the first dashed line) we maximize the extent of P's persistence. The result of the temporal implication rule R1, forces us to clip the persistence of M just after the end of Q. This world will be preferred to any world that is the same as this world except that P stops being true at some point after the end Q due to the persistence assumption: we prefer for P to persist as long as possible. Multiple models, and thus ambiguity or disjunction, result when there are several models none of which is preferred over any of the others. [3] There is a symmetric case, in which M's persistence is maximized. In the second model, the persistence assumptions for P and M are maximized with respect to each other. Neither of the rules come into play in this interpretation. They are maximal with respect to each other in the sense that if you extended either, the others' extent would be reduced. Finally, consider a case where P (or symmetricaly M) is allowed to persist to some point within the extent of Q (N). The third picture shows one of an infinite number of possible worlds that can be obtained in this way. In each of these worlds, the persistence of P and Q are maximized with respect to each other in the same sence as described above.

It is not difficult to come up with similar scenarios involving projection and backward persistence; or temporal implication and forward persistence. In fact fairly complex scenarios can be created using chains of projection rules, temporal implication rules, and persistence. There is an easily-identifiable condition of the causal theory that is necessary but not sufficient condition for theories to entail these kinds of ambiguities. Basically, we look for certain kinds of cycles using static analysis of the rules. Consider a DAG created from the rules as follows:

- Create a node in the DAG for each unique antecedent and consequent proposition
- For each rule create an arc from each antecedent node

[3] For a more careful discussion of the use of model preference to model persistence see *e.g.*, [8; 1]

to the consequent node

- For each consequent node create an arc to each contradictory antecedent

If any cycles exist in this DAG then our theory may entail the kind of non-monotonic disjunction describe above.

We have identified two approaches to implementing a practical system that deals with this kind of disjunction:

- Don't deal with it at all. Use the static rule analysis technique described above to reject rule sets that may entail this kind of disjunction.
- Use an approximation that is sound and incomplete. The idea is to be extremely conservative when looking for possible ambiguities. Any time there is a rule that may participate in a cycle of the sort described above, prohibit any backward persistence from being used as an antecedent.

Both approaches are rather heavy-handed: the analyze-and-complain approach leaves the user either without functionality or without predictability; both approaches overreact to prevent situations that may not occur, on the grounds that specific situation detection is too expensive. This will be a further source of incompleteness in the inference the system does. The complaining approach can be turned into a warning approach that goes on to do weak clipping.

# 5 Summary

In this paper, we have identified the sources of disjunction that must be considered in a temporal reasoning system that handles partially-ordered time points, forward and backward persistence, and two simple forms of causal reasoning. These sources can be grouped roughly into two classes, one corresponding to problems arising from temporal uncertainty (partial orders), the other the result of the nonmonotonic persistence assumption. There is actually a third source of disjunction that we have finessed by restricting the expressive power of the system: we do not permit the expression of explicit disjunctive propositions.

We have demonstrated three general classes of methods for dealing with disjunction, and proposed specific fixes for specific problems. Where possible, we have described implemented solutions from our work on the TMM. This paper presents the first clear characterization of the sources of incompleteness in the sound-and-incomplete decision procedure described in [4].

The techniques we have developed for managing disjunction are crucial to our implementation of an efficient temporal reasoning system. In particular, the representation of a set of disjunctions by some simpler description of a larger set including those disjunctions is a powerful technique that has found repeated use for handling disjunctions with a wide variety of sources and characteristics. With a little care, the resulting system retains the property of soundness, which we regard as crucial to the implementation of a useful system for temporal reasoning.

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